Normal forms and long time existence for semi-linear Klein-Gordon equations

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The aim of the talk will be to describe recent results, concerning the lifetime of smooth solutions of semi-linear Klein-Gordon equations, with small Cauchy data, on some Riemannian manifolds (M, g). If ϵ denotes the size of the data, local existence theory implies that smooth solutions to such equations exist over a time interval of length at least c/ϵ when $\epsilon \to 0+$. Much better lower bounds hold true when $M = \mathbb{R}^d$ and the data are smooth and rapidly decaying at infinity: classical results of Klainerman and Shatah assert that solutions exist then globally in time for small ϵ, when $d \geq 3$. Dispersive properties of the equation – i.e. the fact that solutions of the linear Klein-Gordon equation on $\mathbb{R}^d$, with nice Cauchy data, decay uniformly like $t^{-d/2}$ when $t \to +\infty$ – play an essential role in the proof of such a property.

We shall describe some results obtained over recent years in situations for which such a time decay is not available, like the Klein-Gordon equation on some *compact* manifolds. We shall see that the use of normal form methods allows one to get solutions over long time intervals, in spite of the lack of time decay of linear solutions.