Quasistatic evolution problems in plasticity with softening

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In plasticity theory the term softening refers to the reduction of the yield stress as plastic deformation proceeds. We deal with this problem in the quasistatic case, in the framework of small strain associative elastoplasticity. The presence of a nonconvex term due to the softening phenomenon requires the extension of a variational framework proposed by Mielke to the case of a nonconvex energy functional. In this problem the use of global minimizers in the corresponding incremental problems is not justified from the mechanical point of view. We analize a different selection criterion for the solutions of the quasistatic evolution problem, based on a viscous approximation. In view of the nonconvexity of the problem, taking the limit as the artificial viscosity parameter tends to zero leads to a weak formulation of the problem in a space of Young measures. Moreover, since the growth exponent of the energy is one, we need a suitable notion of generalized Young measure in order to deal with concentration effects. Finally, the classical notion of total variation of a time-dependent function on a time interval has to be extended to time-dependent families of Young measures. This enables us to define, in this generalized context, a notion of dissipation, which plays a crucial role in Mielke's variational approach. Some examples show that smooth initial data may lead, after a critical time, to a Young measure solution with concentration phenomena.