

Contractivity of Wasserstein metrics and asymptotic profiles for nonlinear diffusions and scalar conservation laws

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The aim of this talk is to analyze contractivity properties of Wasserstein-type metrics for general nonlinear diffusions in any dimension and one-dimensional scalar conservation laws and its implications on the long time asymptotics.

We investigate the long time asymptotics in $L^1_+(\mathbf{R})$ for solutions of general nonlinear diffusion equations $u_t = \Delta\phi(u)$. We describe the intermediate asymptotics for a very large class of non-homogeneous nonlinearities ϕ for which long time asymptotics cannot be characterized by self-similar solutions. Scaling the solutions by their own second moment (temperature in the kinetic theory language) we obtain a universal asymptotic profile characterized by fixed points of certain maps in probability measures spaces endowed with the Euclidean Wasserstein distance d_2 .

For scalar conservation laws the flux is assumed to be convex and without any growth condition at the zero state and we work with nonnegative, L^∞ and compactly supported initial data. We propose a time-parameterized family of functions as *intermediate asymptotics* and prove the solutions, after a time-depending scaling, converges towards this family in the d_∞ -Wasserstein metric. This asymptotic behavior relies on the aforementioned contraction property for conservation laws in the space of probability densities metrized with the d_∞ -Wasserstein distance. Finally, we also give asymptotic profiles for initial data whose distributional derivative is a probability measure.

These works are in collaboration with M. Di Francesco and G. Toscani for nonlinear diffusions and with M. Di Francesco and C. Lattanzio for scalar conservation laws.