PLENARY SESSIONS

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Mathematical competence for all: options, implications and obstacles

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Twenty years ago, as a response to the back-to-the-basic movement, the community of mathematics education started to emphasise the need to enlarge the components of what was generally considered as basic skills in mathematics. The inclusion of problem solving, reasoning, applications and the use of technology became major proposals in several programmatic documents.

In the last two decades, the continuous evolution of the society, together with the developments in science, technology and education, led us to consider an increasing number of aspects and to deal with more and more complex problems when discussing what school mathematics for all should be.

Recently, in the context of a movement of educational change in Portuguese basic schools, the need to re-conceive the view about the curriculum led us to consider the concept of "competence" and the process of innovation as major aspects of the movement. With respect to mathematics, a special focus is on the way in which mathematical competence for all may be interpreted and on the corresponding implications for teaching.

The concept of competence may be used (or misused) with several different meanings ranging from a connotation with behaviour and performance to an identification with a quality of a person or a state of being (Short, 1985). In "our" definition, the holistic nature of competence is emphasised. Knowledge is obviously involved, as well as the skill necessary to use it, but this use is an *emancipatory* action, based on reflection and implicating some degree of autonomy. Competence is related to the process of activating resources (knowledge, skills, strategies) in a variety of contexts, namely problematic situations (Perrenoud, 1997).

One of the characteristics of the way in which we view mathematical competence is the *integration* of knowledge, skills, attitudes and beliefs. These are not separate components of the curriculum to be "added" to an organisation traditionally based on specific content topics and routine procedures. Another characteristic is the explicit attention to the nature of mathematics. As Bishop (1991) points out, it is not enough to teach (some) mathematics, it is indeed necessary to educate about, through and with mathematics. In this perspective, mathematical competence cannot be seen in isolation from the educational experiences that all children should live in school, namely investigations and projects involving both mathematical ideas and their relations with different sorts of problems.

Our challenge is indeed to help *all* children to develop their mathematical competence. To do so, however, we have to avoid interpretations reinforcing the perspective of a curriculum of training procedures, skills and rules (for all) with the expectation that this kind of training will constitute (for some) a pre-requisite to future uses of mathematics. Moreover, the formulation of the curriculum should be strongly connected to the purpose of striving against school failure and should take into account all children, namely those with a cultural background not similar to that of the "traditional school".

The adoption of this perspective has consequences both for the kinds of educational experiences to be promoted and for the extension and complexity of topics to be included in the curriculum. But, at the same time, it raises a number of problems and obstacles. Proposals aligned with using and applying mathematics (de Lange, 1996), valuing mathematical investigations (Ernest, 1991) or adhering to the "rebirth" of project teaching (Bishop, 1995), seem to be consistent with the development of mathematical competence in a broad sense. They are often accepted as complementary methods or a sort of "application" but not necessarily as the essence of the curriculum. Clearly, the problem is the resistance to question and abandon the technique-oriented curriculum (Bishop, 1991).
A central aspect of this problem has to do with assessment and control. Popular notions of culture, school and mathematics (Porfírio and Abrantes, 2000), as well as the recent influence of the way in which international comparative studies have been politically interpreted and used (Keitel and Kilpatrick, 1998), tend to reinforce the societal values of competitiveness and standardisation and to favour the perspective of the technique-oriented curriculum. As Keitel (2000) observes, the concept of "globalisation" is ambiguous, having frequently a connotation opposite to the values of cross-country collaboration, interaction and co-operation at different levels.

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From grounding metaphors to technological devices: a call for legitimacy in school mathematics

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Recent research studies have been concerned with the cognitive roots of mathematical concepts. In particular, the work carried out by Lakoff, Johnson and Nunez since the eighties, attempts to build up a new discipline called the cognitive science of Mathematics, based on the notion of embodied cognition. Their basic assumption is that mathematics cannot be considered as mind free: accordingly, they consider the mathematical concepts as coming from the cognitive activities of subjects and highly influenced by the body structure . In short, body is put back into the mind (Johnson, 1987) and language and gestures are given new emphasis (Lakoff and Nunez, 2000). In particular, these authors maintain that metaphors are an essential part of mathematical thought, not just auxiliary mechanisms used for visualization or ease of understanding

Since the beginning of the eighties metaphors have been reconsidered as crucial components of thinking (see Lakoff and Johnson, 1980). The relevance of body-related metaphors in mathematical thinking has been clearly stated with some examples concerning, especially natural numbers and continuity.

Nunez (2000) describes 'conceptual metaphors' as follows: *"It is important to keep in mind that conceptual metaphors are not mere figures of speech, and that they are not just pedagogical tools used to illustrate some educational material. Conceptual metaphors are in fact fundamental cognitive mechanisms (technically, they are inference-preserving cross-domain mappings) which project the inferential structure of a source domain onto a target domain, allowing the use of effortless species-specific body-based inference to structure abstract inference".*

Considering conceptual metaphors, Lakoff and Nunez (2000) make a distinction between 'grounding metaphors' (i. e. metaphors which *"ground our understanding of mathematical ideas in terms of everyday experience"*) and other kinds of metaphors ('Redefinitional metaphors', 'Linking metaphors).

In mathematics as well as in other domains, many metaphors are frequently used with pure communication purposes, especially when mathematicians want to make their comments about mathematical work more impressive. Surely, in a sentence like *"The proof of this theorem was like an obstacle race: once some progress was made, immediately another difficulty appeared"* the 'obstacle race' metaphor fulfils a pure communication purpose which has no mathematical content. 'Communication metaphors' can also be employed in order to substitute technical expressions which are not shared by the interlocutor. This happens frequently in popularisation situation. Another case in which metaphors fulfil a 'substitution' function is when the subject identifies a mathematical object which is not yet covered by a comprehensive definition. These 'substitution' metaphors fulfil a communication purpose by providing some ideas about the mathematical content involved in the discourse.

An interesting question is: “Can metaphors fulfil other functions, in particular 'thinking tool' functions?”

The examples produced by Lakoff and Nunez (2000), show how some metaphors can function as 'thinking tools', in particular as 'ways of thinking' about peculiar mathematical objects. These authors go further in guessing that metaphors (in particular, metaphors referring to everyday body actions and relations) are not exceptions, but usual ways of thinking in mathematics.

The distinction between 'communication metaphors' and 'grounding metaphors', and the very existence of grounding metaphors in a given mathematics domain, are relevant for educational purposes. In the case of grounding metaphors, teachers should legitimate and enhance the use of those metaphors as irreplaceable tools of thinking. Special attention should be paid to the existence and functions of grounding metaphors in those mathematics domains where difficulties in learning are bigger; indeed, ignoring specific grounding metaphors could be one of the reasons for students' difficulties.

In this paper we will discuss the role of grounding metaphors in learning mathematics, by showing how different kinds of grounding metaphors can intervene (as crucial tools of thinking) in novices' approach to inequalities. We will consider a teaching experiment performed in two VIII-grade classes with the main purpose of detecting the young students' potential in dealing with inequalities (Bazzini, Boero and Garuti, 2001) .

The analysis of students' behaviours suggests some possible refinements of the idea of grounding metaphor, and gives evidence of the strong necessity of integrating the embodied approach in a comprehensive educational perspective, in order to make metaphors an object of intentional classroom intervention by the teacher.

At this point another question arises: "Given the theoretical perspective of embodied cognition, do technological devices support mathematics learning?". The very nature of the "digital era" makes this question unavoidable.

To try to find an answer, we will discuss some findings of teaching experiments expressly devoted to the construction and interpretation of graphs coming from collected data by an on-line measurement tool and represented on a graphic calculator. (Arzarello and Robutti, 2001 and Paola, 2001).

There is clear evidence that body, language and instruments mediate and support the transition of students from the perceptual facts to the symbolic representation. The students' cognitive activity is strongly marked by rich language use and gesture activity, for example with production of grounding metaphors.

These research studies are in line with the hypotheses also suggested by Longo (2000) that mathematical concepts are constructed on the base of analogical representation, grounded in our interaction with space and time.

The findings outlined have an important input in school mathematics. They lead to a call for legitimacy for embodied activities, supported also by the use of suitable technological instruments.

This is really a challenge for our future agenda.

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L'enseignement des mathématiques: en regardant le passé, en pensant au futur

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Je me propose de donner un tableau de l'histoire de l'enseignement mathématique en me référant surtout aux élèves de 10-18 ans. Cet enseignement sera considéré dans une contexte social.

L'influence des Eléments d'Euclide traduits en latin et leur introduction dans les écoles des premiers Monastères. La large diffusion de ce traité dans ces écoles après l'invention de l'imprimerie.
Elèves: futurs religieux et enfants de familles riches.
L'incompréhension de la mathématique. Deux témoignages: J.Comenius (1657) et A.C. Clairaut (1741).
La Révolution Française et les articles de la Constitution concernant l'instruction publique.
L'enseignement des mathématiques et l'abstraction comme éléments de séparation entre différents milieux sociaux. Programmes au début du XX siècle, presque égaux dans les différents pays: les comptes-rendus de la CIEM.
Première moitié du XX siècle, sans secousses à part les secousses provoquées par guerres et dictatures.
1959 - Le Séminaire de Royaumont organisé par l'OECE: les "Mathématiques Nouvelles"; ensembles et structures.
1960-'76 - "L'ensemblisme à tout prix", comme disait H. Freudental.
1976 - La prise de position ce M. Atyiah au Congrès de l'ICME a Karlsruhe.
Années '80 : pour une réintroduction de la géométrie. L'influence des programmes italiens pour le premier cycle secondaire.
L'informatique à l'école; faut-il introduire la logique?
Après 1990: groupes d'études internationaux pour l'évaluation.
On observe que bien des élèves, surtout les adolescents, manquent d'intuition et de fantaisie mathématique; pourquoi?
L'espoir qu'on met dans les "nouveaux" élèves qui arrivent de bien des pays extracommunautaires.

Understanding Mathematical Literacy: The Contribution of Research

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Recent efforts to reform school mathematics so that it provides mathematical literacy for all have encountered resistance from some politicians, mathematicians, and the public. Some see school mathematics as a zero-sum game: If your child, with few advantages or inadequate preparation, learns more mathematics, then my child in the same classroom will necessarily learn less. Efforts to improve the mathematics learning of those who have historically had few opportunities are seen as coming at the expense of those who have usually done well in school mathematics. Others worry about the mathematics itself: When mathematics is made relevant to social issues or simply to ordinary daily activities, its vital essences of abstraction, generality, and formality are lost. Curricula that turn from pure to applied mathematics are seen as distorting the subject; students are no longer taught the important mathematics they and society need.

The resulting conflicts have escalated into the so-called math wars in the United States that have pitted the National Council of Teachers of Mathematics and the developers of mathematics curricula against the members of Mathematically Correct and conservative politicians and pundits. In response to disagreements about the direction school mathematics should take, the National Science Foundation and the U.S. Department of Education asked the National Research Council to synthesize the research on pre-kindergarten through eighth-grade mathematics learning to provide recommendations for best practice in the early years of schooling. A 16-member Mathematics Learning Study committee of practitioners, research

mathematicians, cognitive psychologists, researchers in mathematics education, and a representative from the business community met from January 1999 to June 2000; a report of the committee's work was issued this year.
The committee used the term *mathematical proficiency* rather than *mathematical literacy,* but their orientation was much the same as that of the CIEAEM 53 discussion paper. In the presentation, I will discuss how the committee approached its task, how they characterized mathematical proficiency, and how research contributed to the findings and recommendations of the report. I will also discuss how the report provided a much-needed "middle ground" amid the torrent of imprecations and manifestos roiling current discourse about school mathematics.

For a Learnable Mathematics in the Computational Era

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The appearances of new computational forms are pervading the social and economic lives of individuals and nations alike. Yet nowhere is this upheaval correspondingly represented in educational systems or in classrooms. As far as mathematics is concerned, the changes that characterize the late twentieth century—in terms of the way mathematics is done, and what counts as mathematics—are almost invisible in the classrooms of our schools and, to only a slightly lesser extent, our universities.
The real changes are not merely technical: they are cultural. Understanding them is a question of the social relations among people, not merely among things. There are important ways in which computational technologies are different from those that preceded them, and we should attempt to assess the actual and potential contribution of these technologies to mathematical education. From a cultural point of view, we need to consider the ways in which mathematical relations are represented, their social functioning, and the ways in which new ways of expressing mathematical ideas (and new mathematical ideas) are reshaping the infrastructures available for individuals and social groups to interpret the world and, perhaps, to change it.
The possibilities inherent in emergent representational infrastructures are at once exciting and challenging. They raise difficult questions about epistemology, about learnability, and about equity. In particular, a central challenge of mathematical learning for educators is the design of learnable systems, which depend for their learnability on fine-grained specificities of the systems themselves, as well as the broader cultural roles they play in shaping what is possible and natural to express.
These questions are most often thought of as causal in nature: either technology is seen as a cause of educational change or indifference, or it is viewed as a necessary corollary of didactical or societal pressures. But one-way causality overlooks the importance of the interrelationships between technologies, epistemologies and didactics, and leads, for example, to impatience with the slowness with which technology appears to bring about educational transformation. An alternative viewpoint regards technological infrastructure and didactical practice as coevolutionary partners in the reshaping of educational cultures.
The development of mathematical notations (such as algebra, calculus, epsilon-delta and so on) were designed to afford fluency among an intellectual élite, and more or less without regard to their learnability (although, as diSessa, 2000, points out, some notations triumphed over others precisely because they were more learnable than their competitors). All such structures arose within the semiotic constraints of static, inert media. Over the past several centuries these intellectual tools, methods and products (the foundations of the science and technology upon which we depend) were not only institutionalized as the structure and core content of school and

university curricula in most industrialized countries but were often taken as the epistemological essence of mathematics and the yardstick against which academic success was defined. Thus the close relationship of knowledge and its privileged representations, precisely the coupling that has produced such a powerful synergy for developing scientific ideas since the Renaissance, became at the same time an obstacle to learning, and even a barrier which prevented whole classes from accessing the ideas which the representations were so finely tuned to express (see Kaput, Hoyles and Noss, in press, for an elaboration of the point).

While the execution of processes was necessarily subsumed within the individual mind, decoupling knowledge from its preferred representation was difficult[1]. But digital technologies have radically changed this situation. The emergence of what Kaput & Shaffer (in press) call a "virtual culture" has had far-reaching implications for what it is that people need to know, as well as how they can express that knowledge. We may, in fact, have to reevaluate what knowledge itself is, now that knowledge *and the means to act on it* can exist independently from human agency.

The systems that now control our lives are built on mathematical principles. Algorithms, and their instantiation in computer programs, are now a ubiquitous form of knowledge, and they—or at least the outcomes of their execution—are fundamental to the working and recreational experiences of all individuals within the developed world. This is a major—perhaps *the* major—property of the virtual culture. The devolution of execution to the machines means not only that the machines now *do* mathematical execution, but that any consequential appreciation of what the machines do must itself be based on mathematical principles. If an individual does not have the means formally to relate his or her intellectual model of the mathematical principles with those inside the machine, then appreciation of the model must necessarily be partial.

The widespread appearance of these forms of knowledge has not been without cost to individuals and social groups. Many individuals and social groups have suffered a massive deskilling of their working lives precisely because of this devolution of executive power to the machine. One might be forgiven for believing that this shift from human to machine processing removes the necessity for human expression and interpretation altogether[2]. But this fallacy falls victim to the assumption of one-way causality. In reality, the situation is considerably more complex, and in order to understand it, we will need to look more closely at the new ways in which mathematics enters into our cultures, and the ways in which it is represented and transformed as it does so. Representation is not passive.

In fact, the devolution of processing power to the computer has generated the need for a new intellectual infrastructure; people need to represent for themselves how things work, what makes systems fail and what would be needed to correct them. This kind of knowledge is increasingly important; it is knowledge that potentially unlocks the mathematics that is wrapped invisibly into the systems we now use, and yet understand so little of. Increasingly, we need—to put it bluntly—to *make sense of mechanism*. To make this case, I will draw on a series of studies that I and my colleagues have undertaken in London with a range of professions (aviation pilots, nurses, bank employees, and most recently, engineers).

These new roles for mathematical activities and expression will suggest a vision of a mathematical literacy that is based on the construction and interpretation of quantitative and semi-quantitative models, where students explore mathematical technologies and analyze methods in contexts that show how they can be used and why they work in the way they do. This is a mathematics which can, perhaps for the first time, make moves to span the hitherto unbridgeable gulf between mathematics as a discipline and as a set of tools, between theory and application. The examples I will present vision will largely be drawn from recent work which

[1] Though not impossible: a considerable amount of mathematical education research has tried to study—and encourage—the ways in which people form conceptual images of mathematical ideas independently of—and sometimes in conflict with—the preferred algebraic or formal representation.

[2] Some have celebrated this 'liberation' from the need to know by suggesting that here is no need for 'users' to know any non-trivial mathematics themselves (see, for example, Brammall and White, 2000. *Why Learn Maths?* Bedford Way Papers, Institute of Education).

Hoyles and myself have undertaken with extremely young children (see www.ioe.ac.uk/playground).

Finally, I will argue that computational media afford the opportunity to create democratizing infrastructures which will redefine school knowledge (these issues are discussed in depth in Noss and Hoyles, 1996). Viewed optimistically, these will exploit the processing power of the new media to ensure that that students maintain an intuitive feel for the central knowledge elements at work and how they relate to each other and develop together. This will form an important step away from a 19th century school mathematics that concentrates on isolated skills based on static representational systems in a tightly-defined curriculum, in which only a minority are able to engage in critical or expressive activity. Understanding what it may be a step towards, remains the key design challenge for both mathematicians and educators.

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